

UCC5C — Resource Management Techniques

Theory Questions & Answers

BCA — Semester 5 | Based on University Question Papers 2020–2024

SECTION A — 2 Mark Questions

Operations Research (O.R.)

Q1. [2 Marks] (Asked: Dec2020, Dec2024, May2021, May2024)

What is Operations Research (O.R.)?

Answer:

Operations Research (O.R.) is a scientific and quantitative approach to decision-making. It applies mathematical, statistical, and analytical methods to find the best (optimal) solution to complex managerial and organizational problems. O.R. helps in planning, scheduling, and resource allocation across industries such as manufacturing, logistics, healthcare, and finance.

Q2. [2 Marks] (Asked: May2021, May2024)

What are the necessity / applications of O.R.?

Answer:

- Helps in optimal allocation of limited resources such as manpower, machines, and money.
- Supports effective decision-making in complex business situations.
- Used in transportation, inventory control, project scheduling, and production planning.
- Enables managers to evaluate multiple alternatives and choose the best course of action.

Q3. [2 Marks] (Asked: Dec2021)

What are two limitations of an O.R. model?

Answer:

- O.R. models are based on mathematical assumptions that may not reflect real-world complexity.
- Large-scale O.R. problems require significant computational resources and expertise to solve.

Q4. [2 Marks] (Asked: May2022, Dec2023)

What are the objectives of Operations Research?

Answer:

- To provide a scientific basis for decision-making by using quantitative models.
- To find optimal solutions to problems involving allocation of scarce resources.
- To reduce costs and maximize efficiency in organizational operations.

Linear Programming Problem (LPP)

Q5. [2 Marks] (Asked: Dec2022, Dec2023, May2023, May2024)

What is LPP (Linear Programming Problem)? / Define Linear Programming.**Answer:**

A Linear Programming Problem (LPP) is a mathematical optimization technique used to maximize or minimize a linear objective function subject to a set of linear constraints and non-negativity restrictions. It is widely used in production planning, resource allocation, diet problems, and transportation.

Q6. [2 Marks] (Asked: Dec2020, Dec2024)

What is Simplex Method?**Answer:**

The Simplex Method is an iterative algebraic algorithm for solving Linear Programming Problems with two or more decision variables. Starting from an initial basic feasible solution (a corner point of the feasible region), it moves step by step to adjacent corner points that improve the objective function, until the optimal solution is reached.

Q7. [2 Marks] (Asked: Dec2020, Dec2024)

What is Graphical Method?**Answer:**

The Graphical Method is a technique for solving LPPs with exactly two decision variables. Constraints are plotted as lines on a graph; the feasible region is identified; corner points of the feasible region are evaluated in the objective function; and the corner yielding the maximum or minimum value is the optimal solution.

Q8. [2 Marks] (Asked: Dec2020, Dec2021)

What are the characteristics of Standard Form of LPP?**Answer:**

- The objective function is to be maximized (or converted to maximization).
- All constraints are expressed as equalities by adding slack/surplus/artificial variables.
- All variables (decision and slack) are non-negative.
- The right-hand side (RHS) of each constraint is non-negative.

Q9. [2 Marks] (Asked: Dec2020, Dec2023)

What is Primal Problem? / What is Duality?**Answer:**

The Primal Problem is the original formulation of an LPP with its own objective function and constraints. Every primal LPP has a corresponding Dual Problem: if the primal is a maximization problem, the dual is a minimization problem (and vice versa). The optimal value of both the primal and dual objective functions is identical — this is known as the Duality theorem.

Q10. [2 Marks] (Asked: May2022, May2023)

What are Slack and Surplus Variables? / What is a Feasible Solution?**Answer:**

- **Slack Variable:** Added to a $\leq$ constraint to convert it to equality. It represents unused capacity (e.g., $x_1 + x_2 + s = 10$).
- **Surplus Variable:** Subtracted from a $\geq$ constraint to convert it to equality. It represents the excess over the minimum requirement.
- **Feasible Solution:** Any set of values for the decision variables that satisfies all constraints including non-negativity conditions.

Q11. [2 Marks] (Asked: Dec2022, May2023)

What is an Artificial Variable?**Answer:**

An Artificial Variable is a non-negative variable added to a constraint (of $\geq$ or $=$ type) to obtain an initial basic feasible solution in the Big-M method or Two-Phase simplex method. It has no physical meaning and must be driven to zero before the optimal solution is accepted. A very large penalty cost (M) is assigned to it in the objective function to ensure it leaves the basis.

Q12. [2 Marks] (Asked: May2023)

What is Two-Phase Method?

Answer:

The Two-Phase Method is an alternative to the Big-M method for solving LPPs involving artificial variables. Phase 1: Minimize the sum of all artificial variables; if the minimum is zero, a basic feasible solution exists. Phase 2: Use the feasible basis from Phase 1 and optimize the original objective function using the simplex method.

Q13. [2 Marks] (Asked: May2023)

What is Canonical Form of LPP?

Answer:

A canonical form of LPP (for maximization) requires: (a) the objective function is to be maximized, (b) all constraints are of $\leq$ type, and (c) all variables are non-negative. For minimization problems, the canonical form requires $\geq$ constraints. It simplifies writing the dual and applying the simplex algorithm.

Transportation Problem

Q14. [2 Marks] (Asked: Dec2021, Dec2022, Dec2023, May2022)

What is Transportation Problem? / What are Transportation Models?

Answer:

A Transportation Problem is a special type of Linear Programming Problem concerned with finding the minimum-cost plan for shipping goods from multiple sources (factories/warehouses with given supply) to multiple destinations (markets/stores with given demand). It forms a balanced problem when total supply = total demand; otherwise a dummy row or column is added.

Q15. [2 Marks] (Asked: May2023)

What is an Unbalanced Transportation Problem?

Answer:

A Transportation Problem is said to be unbalanced when the total supply does not equal the total demand. If supply > demand, a dummy destination column with zero transportation cost is added to absorb the excess supply. If demand > supply, a dummy source row with zero cost is added.

Assignment Problem

Q16. [2 Marks] (Asked: Dec2020, Dec2021, Dec2023, Dec2024, May2022)

What is Assignment Problem?

Answer:

An Assignment Problem is a special case of the Transportation Problem where n tasks (jobs) are assigned to n agents (workers/machines) on a one-to-one basis such that the total cost (or time) is minimized. Each agent performs exactly one job and each job is done by exactly one agent. It is solved using the Hungarian Method.

Q17. [2 Marks] (Asked: Dec2024)

What is Assignment Model?**Answer:**

An Assignment Model is a mathematical framework for the optimal one-to-one matching of resources to tasks. It assumes a square cost matrix ($n \times n$), one assignment per row and column, and seeks to minimize total assignment cost. If the matrix is unbalanced (unequal rows and columns), a dummy row or column with zero costs is added.

Q18. [2 Marks] (Asked: Dec2022)

When is an Assignment Problem Unbalanced?**Answer:**

An Assignment Problem is unbalanced when the number of jobs (rows) is not equal to the number of machines or workers (columns) — i.e., the cost matrix is not square. To solve it, dummy rows or columns with zero cost values are added to make it a square matrix before applying the Hungarian Method.

Q19. [2 Marks] (Asked: Dec2021, May2021)

What is the Travelling Salesman Problem?**Answer:**

The Travelling Salesman Problem (TSP) asks: given a list of cities and the distances between them, what is the shortest possible route that visits each city exactly once and returns to the starting city? It is a combinatorial optimization problem and an extension of the Assignment Problem. TSP is NP-hard in general but can be solved for small instances using assignment or branch-and-bound techniques.

Game Theory

Q20. [2 Marks] (Asked: Dec2020, Dec2021, Dec2022, Dec2023, Dec2024, May2022)

What is Game Theory? / Short note on Game Theory.**Answer:**

Game Theory is a branch of Operations Research that deals with decision-making in competitive situations involving two or more players (decision-makers) with conflicting interests. Each player chooses a strategy to maximize their own gain (or minimize loss), knowing the opponent will act similarly. It is widely used in economics, military strategy, business competition, and auction design.

- Key terms: Players, Strategies, Pay-off, Saddle Point, Value of the Game.

Q21. [2 Marks] (Asked: Dec2022, Dec2024)

What is a Pay-off Matrix?**Answer:**

A Pay-off Matrix is a table that shows the outcomes (gains or losses) for each combination of strategies chosen by the players in a game. Rows represent Player A's strategies and columns represent Player B's strategies. Each cell entry is the gain to Player A (and loss to Player B in a zero-sum game).

Q22. [2 Marks] (Asked: Dec2021)

What is a Zero-Sum Game?**Answer:**

A Zero-Sum Game is a competitive situation in which one player's gain is exactly equal to the other player's loss, so the total payoff always sums to zero. Most two-player games studied in OR are zero-sum games. Example: If Player A gains ₹5, Player B loses ₹5, and the net sum = 0.

Network Analysis — PERT & CPM

Q23. [2 Marks] (Asked: Dec2020, Dec2021, Dec2022, Dec2023, Dec2024, May2022, May2023)

What is PERT? (Write the expansion of PERT)

Answer:

PERT stands for Programme Evaluation and Review Technique. It is a probabilistic network-based project management technique developed by the U.S. Navy in 1958 for the Polaris missile project. PERT is used when activity durations are uncertain; it uses three time estimates — optimistic (to), most likely (tm), and pessimistic (tp) — to compute expected time:

$$te = (to + 4tm + tp) / 6$$

Q24. [2 Marks] (Asked: Dec2020, Dec2021, Dec2024, May2021)

What is CPM? / Write any two advantages of CPM.

Answer:

CPM stands for Critical Path Method. It is a deterministic network technique used when activity durations are fixed and known. CPM identifies the critical path — the longest path through the network — which determines the minimum project duration. Any delay in critical activities directly delays the project.

Advantages of CPM:

- Identifies the critical activities that need close monitoring and control.
- Helps in time-cost trade-off analysis — activities can be crashed (shortened) by spending more resources.
- Provides a clear visual network showing dependencies between activities.

Q25. [2 Marks] (Asked: Dec2022)

What is a Dummy Activity?

Answer:

A Dummy Activity is a fictitious activity in a PERT/CPM network that consumes zero time and zero resources. It is represented by a dashed arrow and is used to: (a) maintain the logical sequence of activities, and (b) avoid giving the same i-j numbering to two different activities (to maintain the uniqueness of event numbers). It shows dependency without any real work.

Q26. [2 Marks] (Asked: Dec2022)

What is the Critical Path?

Answer:

The Critical Path is the longest continuous path from the start event to the end event in a project network. All activities on this path have zero float (slack), meaning any delay in them directly extends the overall project duration. The length of the critical path equals the minimum project completion time.

Q27. [2 Marks] (Asked: Dec2022)

State any two Network Techniques.

Answer:

- CPM (Critical Path Method) — deterministic; used for well-defined, repetitive projects.
- PERT (Programme Evaluation and Review Technique) — probabilistic; used for R&D and uncertain projects.

Sequencing Problem

Q28. [2 Marks] (Asked: Dec2021, May2022, May2024)

What is a Sequencing Problem?

Answer:

A Sequencing Problem involves determining the optimal order in which a set of jobs should be processed through a series of machines to minimize total elapsed time (or idle time). For example, in Johnson's algorithm for n jobs on 2 machines: list jobs with minimum processing time on Machine 1 first and jobs with minimum on Machine 2 last.

Q29. [2 Marks] (Asked: Dec2021)

What is Idle Time on Machines?

Answer:

Idle Time on a machine is the period during which the machine is available but has no job to process. It occurs when the next job in sequence has not finished its processing on the previous machine. Minimizing idle time is one of the key objectives of sequencing problems, as idle time represents wasted machine capacity.

Simulation & Monte Carlo

Q30. [2 Marks] (Asked: Dec2021, Dec2022, Dec2023, May2022)

What is Monte Carlo Method / Simulation?

Answer:

Monte Carlo Simulation is a technique that uses random numbers to simulate the behaviour of a real-world system over time. It is applied when a system is too complex for analytical solution. Random numbers are used to generate values of uncertain variables (e.g., demand, service time) based on probability distributions, and the simulation is run many times to observe average outcomes.

Q31. [2 Marks] (Asked: Dec2022)

What is a Random Number?

Answer:

A Random Number is a number chosen from a set such that each number has an equal probability of being selected, with no pattern or predictability. In Monte Carlo simulation, random numbers (typically between 0 and 99, or 0.00 and 0.99) are used to assign values to probabilistic variables by mapping them to cumulative probability distributions.

Q32. [2 Marks] (Asked: May2021)

What is Computer Simulation?

Answer:

Computer Simulation is the use of a computer program to model and imitate the operations of a real-world system over time. It allows testing different scenarios and policies without physically implementing them. In OR, it is used to analyze queuing systems, inventory systems, and project risk, especially when the problem is too complex for mathematical optimization.

Q33. [2 Marks] (Asked: May2024)

What are the uses / advantages of Computer Simulation / Monte Carlo Method?

Answer:

- Allows analysis of complex systems that cannot be solved analytically.
- Can model uncertainty and variability in input data using probability distributions.
- Helps in testing various policies and strategies without real-world risk.
- Useful for forecasting outcomes and understanding system behaviour.

SECTION B — 5 Mark Questions

Operations Research — Scope, Characteristics & Decision Making

Q34. [5 Marks] (Asked: Dec2023, May2021, May2022, Dec2024)

Explain the Scope / Characteristics of Operations Research.

Answer:

Operations Research has a broad scope spanning nearly every industry and management function. Its key areas include:

- **Resource Allocation:** Optimal distribution of limited resources (machines, manpower, money) across competing activities — e.g., production planning.
- **Inventory Management:** Determining when and how much to order/produce to balance holding and ordering costs (EOQ, ABC analysis).
- **Transportation & Distribution:** Finding least-cost shipping plans from factories to warehouses to customers.
- **Project Management:** Planning and scheduling complex projects using CPM and PERT networks.
- **Queuing / Waiting Lines:** Analyzing service systems to minimize waiting time and optimize server capacity.
- **Game Theory:** Modeling competitive decision-making in markets, bidding, and strategic planning.
- **Simulation:** Imitating complex systems to test policies when direct analysis is infeasible.

Characteristics of O.R.: It is inter-disciplinary, uses mathematical models, requires team approach, aims for optimal decisions, and is computer-assisted for large problems.

Q35. [5 Marks] (Asked: Dec2023)

What are the needs and significance of Operations Research?

Answer:

- **Growing Complexity:** Modern organizations are too complex for intuitive management; OR provides systematic tools.
- **Resource Scarcity:** Helps allocate scarce resources optimally to maximize output or minimize cost.
- **Competitive Pressure:** OR-based decisions give organizations a competitive edge in pricing, production, and logistics.
- **Risk Reduction:** Quantitative analysis reduces guesswork and uncertainty in managerial decisions.
- **Technology Integration:** OR models can be implemented via computers and ERP systems for real-time optimization.

Duality in LPP

Q36. [5 Marks] (Asked: Dec2021, May2022)

Describe the general rules for writing the Dual of a LPP.

Answer:

Every LPP (primal) has a corresponding dual. The rules are:

- If the primal is a Maximization problem, the dual is a Minimization problem (and vice versa).
- The number of variables in the dual = the number of constraints in the primal.
- The number of constraints in the dual = the number of variables in the primal.
- The objective function coefficients of the primal become the RHS of the dual constraints.
- The RHS values of the primal constraints become the objective function coefficients of the dual.
- The coefficient matrix of the dual is the transpose of the primal constraint matrix.

- $A \leq$ constraint in a maximization primal $\rightarrow$ the corresponding dual variable ≥ 0 . $A \geq$ constraint $\rightarrow$ dual variable ≥ 0 (for minimization primal). $A =$ constraint $\rightarrow$ dual variable is unrestricted.
- An unrestricted variable in the primal corresponds to an equality constraint in the dual.

Key theorem: The optimal values of the primal and dual objective functions are equal.

Q37. [5 Marks] (Asked: May2022)

Write the properties of Primal and Dual Optimal Solutions.

Answer:

- **Strong Duality:** If the primal has an optimal solution, so does the dual, and both have the same optimal objective value.
- **Complementary Slackness:** At optimality, if a primal variable > 0 , the corresponding dual constraint is an equality; if a dual variable > 0 , the corresponding primal constraint is an equality.
- **Weak Duality:** Any feasible solution to the dual provides a bound on the primal: for maximization primal, dual objective $\geq$ primal objective.
- **Infeasibility:** If the primal is infeasible, the dual is either infeasible or unbounded.
- **Unboundedness:** If the primal is unbounded, the dual is infeasible.

Assignment Problem — Characteristics & Model

Q38. [5 Marks] (Asked: Dec2020, Dec2023, Dec2024, May2021)

Discuss any three characteristics of Assignment Model.

Answer:

- **One-to-One Assignment:** Each job (task) is assigned to exactly one machine (worker) and each machine handles exactly one job. No sharing is allowed.
- **Square Matrix:** The assignment problem requires a square cost matrix ($n \times n$). If the numbers of jobs and machines differ, dummy rows or columns with zero cost are added.
- **Minimization Objective:** The standard assignment problem seeks to minimize total cost (time, distance, etc.). For maximization, convert by subtracting each element from the maximum element.
- **Hungarian Method:** The assignment problem is solved optimally and efficiently using the Hungarian Algorithm, which operates through row reduction, column reduction, and line-covering steps.
- **Binary Decision:** Each decision variable is 0 or 1 (assign or not assign), making it a special integer programming problem.

Travelling Salesman Problem

Q39. [5 Marks] (Asked: Dec2020, Dec2023, Dec2024, May2021)

Discuss the concept / nature of the Travelling Salesman Problem (TSP).

Answer:

The Travelling Salesman Problem (TSP) is one of the most famous combinatorial optimization problems. A salesman must visit n cities exactly once, starting and ending at the home city, using the route of minimum total distance (or cost).

- **Nature:** It is a special case of the Assignment Problem where: (a) every city must be visited, (b) no city is revisited, and (c) the tour must form a single connected loop (no sub-tours).
- **Formulation:** Set up an assignment matrix with infinite cost (∞) on the diagonal (city to itself). Solve the assignment problem; if the solution contains sub-tours, add sub-tour elimination constraints.
- **Complexity:** TSP is NP-hard: the number of possible routes is $(n-1)!/2$, which grows exponentially with n .

- **Applications:** Vehicle routing, delivery scheduling, circuit board drilling, DNA sequencing, and network design.
- **Solution Methods:** Exact: Branch-and-Bound, Dynamic Programming. Approximate: Nearest Neighbour heuristic, Genetic Algorithms.

Transportation — Vogel's Approximation Method (VAM)

Q40. [5 Marks] (Asked: Dec2020, Dec2024, May2024)

Explain Vogel's Approximation Method (VAM) in detail.

Answer:

VAM is an improved method for finding the initial basic feasible solution of a transportation problem. It generally gives a solution very close to the optimal (often optimal itself).

- **Step 1:** Calculate the penalty for each row and column: penalty = difference between the two smallest costs in that row/column.
- **Step 2:** Select the row or column with the highest penalty. Allocate as much as possible to the cell with the lowest cost in that row/column. If there is a tie in penalties, choose the one with the lower minimum cost.
- **Step 3:** Adjust supply and demand. If supply is exhausted, eliminate the row. If demand is satisfied, eliminate the column. If both are exhausted, eliminate one (and record a zero allocation in the other if needed).
- **Step 4:** Recalculate penalties for remaining rows and columns after each allocation.
- **Step 5:** Repeat Steps 2–4 until all supply is allocated and all demand is satisfied.

VAM is preferred over the North-West Corner and Least Cost methods because it considers the cost difference (opportunity cost) in making allocations, leading to a better starting solution.

Game Theory — Characteristics & Solving 2×2 Games

Q41. [5 Marks] (Asked: May2022)

Write the characteristics of Game Theory.

Answer:

- **Number of Players:** A game involves two or more players (decision makers). Two-player games are most commonly studied in OR.
- **Finite Strategies:** Each player has a finite number of strategies to choose from.
- **Pay-off Matrix:** The outcome of every combination of strategies is represented in a pay-off matrix.
- **Zero-Sum Property:** In a two-person zero-sum game, the gain of one player equals the loss of the other.
- **Rationality:** All players are assumed to be rational — they always choose the strategy that maximizes their own gain.
- **Complete Information:** Each player knows the pay-off matrix and the strategies available to all players.
- **Saddle Point:** If the maximin (max of row minima) equals the minimax (min of column maxima), a saddle point (pure strategy solution) exists and the game is strictly determined.
- **Mixed Strategies:** If no saddle point exists, players randomize over strategies with certain probabilities — called mixed strategies.

Q42. [5 Marks] (Asked: Dec2021)

Solve the 2×2 game using Mixed Strategy method. (General Method)

Answer:

For a 2×2 game with pay-off matrix:

B1 B2

A1 a b

A2 c d

First check for a saddle point (maximin = minimax). If none exists, use mixed strategies:

- **Player A's optimal probabilities:** $p_1 = (d - c) / (a - b - c + d)$, $p_2 = 1 - p_1$
- **Player B's optimal probabilities:** $q_1 = (d - b) / (a - b - c + d)$, $q_2 = 1 - q_1$
- **Value of the Game:** $V = (ad - bc) / (a - b - c + d)$

These formulas arise from the condition that each player's expected gain must be equal regardless of the opponent's strategy — the principle of indifference.

Network Analysis — CPM Steps & Network Drawing

Q43. [5 Marks] (Asked: Dec2022)

Describe the steps in CPM.

Answer:

- **Step 1 – List Activities:** Identify all activities required to complete the project, their durations, and dependencies.
- **Step 2 – Draw Network:** Construct the arrow diagram (network) with activities as arrows and events (milestones) as nodes. Assign unique node numbers.
- **Step 3 – Forward Pass:** Compute Earliest Event Time (E) for each node from left to right: $E(j) = \max[E(i) + \text{duration}(i-j)]$.
- **Step 4 – Backward Pass:** Compute Latest Event Time (L) for each node from right to left: $L(i) = \min[L(j) - \text{duration}(i-j)]$.
- **Step 5 – Compute Float:** Total Float = $L(j) - E(i) - \text{duration}(i-j)$. Activities with Float = 0 are critical.
- **Step 6 – Critical Path:** Connect all critical activities (zero float) from start to end — this is the critical path and determines the project duration.

Monte Carlo Simulation

Q44. [5 Marks] (Asked: Dec2020, Dec2021, Dec2023, Dec2024, May2021)

Explain the steps involved in Monte Carlo Simulation.

Answer:

- **Step 1 – Identify the Problem:** Define the system or process to be simulated and the variable(s) whose behaviour is uncertain (e.g., demand, service time).
- **Step 2 – Determine Probability Distribution:** Based on historical data or expert knowledge, construct the probability distribution for each uncertain variable.
- **Step 3 – Build Cumulative Distribution:** Convert the probability distribution to a Cumulative Probability Distribution (CDF).
- **Step 4 – Assign Random Number Intervals:** Map random numbers (00–99) to the cumulative distribution: each value of the variable gets a range of random numbers proportional to its probability.
- **Step 5 – Generate Random Numbers:** Use a random number table or random number generator to obtain a sequence of random numbers.
- **Step 6 – Simulate the System:** For each random number, read the corresponding variable value from the mapping table. Record outcomes over many trials.
- **Step 7 – Analyse Results:** Compute averages, totals, or other statistics from the simulated data to draw conclusions about the real system's behaviour.

Monte Carlo simulation is particularly useful for inventory, queuing, and risk analysis problems where analytical solutions are difficult to obtain.

SECTION C — 10 Mark Questions

Simplex Method — Detailed Theory

Q45. [10 Marks] (Asked: Dec2020, Dec2024)

Explain the Simplex Method of solving a Linear Programming Problem.

Answer:

The Simplex Method, developed by George Dantzig in 1947, is the standard algorithm for solving LPPs with any number of variables. It works by moving from one corner point (basic feasible solution) of the feasible region to an adjacent better corner until the optimal solution is found.

Step-by-Step Procedure:

- **Step 1 – Standard Form:** Convert the LPP to standard form: add slack variables ($s_1, s_2, \dots$) to $\leq$ constraints, subtract surplus and add artificial variables to $\geq$ constraints.
- **Step 2 – Initial Tableau:** Set up the initial Simplex tableau with the basic variables (slack/artificial), coefficients, and RHS values.
- **Step 3 – Optimality Check:** For maximization: if all coefficients in the objective row (Z-row) are ≥ 0 , the current solution is optimal. Otherwise, continue.
- **Step 4 – Identify Entering Variable:** Choose the variable with the most negative coefficient in the Z-row as the entering variable (it gives the greatest improvement).
- **Step 5 – Identify Leaving Variable:** Apply the Minimum Ratio Test: divide each positive element in the pivot column into the corresponding RHS value. The row with the smallest non-negative ratio gives the leaving variable.
- **Step 6 – Pivot Operation:** Perform row operations: divide the pivot row by the pivot element to make it 1; then eliminate all other entries in the pivot column by row subtraction.
- **Step 7 – Update and Repeat:** Update the Z-row and basic variable column. Return to Step 3 and repeat until optimal.

Special Cases: (a) If two rows tie in Step 5, degeneracy occurs — choose arbitrarily. (b) If no positive element exists in the pivot column, the problem is unbounded.

Duality — Theory & Solving via Dual

Q46. [10 Marks] (Asked: Dec2023, Dec2024, May2021)

Use duality and solve a LPP. / Discuss the concept of duality in detail.

Answer:

Duality is one of the most elegant and useful concepts in Linear Programming. Every LPP (called the Primal) has an associated LPP (called the Dual). Solving one automatically gives the solution to the other.

Economic Interpretation:

In a production problem, the primal variables represent quantities produced and the dual variables represent 'shadow prices' — the marginal value of each unit of a resource. If a resource has positive shadow price, it is binding (fully utilized); if zero, it has surplus.

Rules for Writing the Dual:

- Primal: Maximize $\rightarrow$ Dual: Minimize (and vice versa).
- Primal variables (n) $\rightarrow$ Dual constraints (n). Primal constraints (m) $\rightarrow$ Dual variables (m).
- Primal objective coefficients $\rightarrow$ Dual RHS. Primal RHS $\rightarrow$ Dual objective.
- Transposed coefficient matrix in dual constraints.

Key Theorems:

- **Weak Duality:** Any dual feasible solution provides an upper bound (for minimization) to the primal optimal.
- **Strong Duality:** If primal has optimal solution, dual has same optimal value.
- **Complementary Slackness:** Primal variable $\times$ dual slack = 0, and dual variable $\times$ primal slack = 0 at optimality.

Method — Solving via Dual:

- Write the dual of the given primal.
- Solve the dual using the simplex method.
- Read the primal optimal solution from the dual simplex tableau (values of primal variables appear in the Z-row under the slack variable columns).

Transportation vs Assignment Problem

Q47. [10 Marks] (Asked: Dec2020, Dec2024)

Explain the difference between a Transportation Problem and an Assignment Problem.

Answer:

Although both are special cases of LPP dealing with allocation, they differ significantly:

Transportation Problem:

- Deals with shipping goods from m sources to n destinations.
- Supply at each source and demand at each destination can be any positive value.
- Multiple units can be shipped from one source to one destination.
- Objective: minimize total transportation cost.
- Solved using: North-West Corner, Least Cost, VAM (initial solution) + MODI/Stepping Stone (optimality).
- Matrix need not be square.

Assignment Problem:

- Deals with assigning n jobs to n workers (or machines) on a one-to-one basis.
- Each agent handles exactly one job; each job is done by exactly one agent.
- Supply and demand are each equal to 1 for every source and destination.
- Objective: minimize total assignment cost (or maximize profit).
- Solved using: Hungarian Method.
- Matrix must be square ($n \times n$); dummy rows/columns added if unbalanced.

Summary of Differences:

- Assignment is a special case of transportation where all supplies and demands = 1.
- Transportation can have fractional allocations; assignment is binary (0 or 1).
- Transportation may have multiple optimal solutions; assignment gives a unique optimal set of assignments.

PERT Estimation Techniques — Detailed

Q48. [10 Marks] (Asked: Dec2022, Dec2023, Dec2024, May2021)

Discuss in detail about PERT Estimation Techniques.

Answer:

PERT (Programme Evaluation and Review Technique) was developed in 1958 by the U.S. Navy for managing the Polaris missile project. Unlike CPM, PERT treats activity durations as probabilistic, making it suitable for research and development projects where durations are uncertain.

Three-Time Estimation:

- **Optimistic Time (to):** Minimum time to complete an activity if everything goes perfectly well.
- **Most Likely Time (tm):** The most realistic estimate under normal working conditions.
- **Pessimistic Time (tp):** Maximum time if everything goes wrong (worst-case scenario).

Expected Time Formula (Beta Distribution):

$$t_e = (t_o + 4t_m + t_p) / 6$$

The factor of 4 gives the greatest weight to the most likely estimate.

Variance and Standard Deviation:

$$\text{Variance } \sigma^2 = [(t_p - t_o) / 6]^2$$

$$\text{Standard Deviation } \sigma = (t_p - t_o) / 6$$

Project Expected Duration and Variance:

Project $T_e = \sum t_e$ of critical path activities

Project $\sigma^2_p = \sum \sigma^2$ of critical path activities

Probability of Completion:

Using the normal distribution, the probability of completing the project by a scheduled date T_s :

$$Z = (T_s - T_e) / \sigma_p$$

Look up Z in the standard normal table to find the probability $P(T \leq T_s)$.

Advantages of PERT:

- Accounts for uncertainty in activity durations.
- Allows probability statements about project completion.
- Identifies critical activities for management focus.
- Useful for new, complex, one-time projects.

Limitations of PERT:

- Requires three time estimates, which are often subjective.
- Assumes Beta distribution for activity times — may not always be appropriate.
- Does not consider resource constraints.

Game Theory — Detailed Theory & Mixed Strategies

Q49. [10 Marks] (Asked: May2022, Dec2022, May2023)

Solve the game whose pay-off matrix is given. / Theory of Games — Saddle Point & Mixed Strategy.

Answer:

Game Theory provides a mathematical framework for analyzing competitive situations. Key concepts:

Pure Strategy & Saddle Point:

- **Maximin Strategy (Player A):** For each row, find the minimum pay-off. Player A chooses the row with the maximum of these minima (best of worst cases).
- **Minimax Strategy (Player B):** For each column, find the maximum pay-off. Player B chooses the column with the minimum of these maxima.
- **Saddle Point:** If Maximin = Minimax, the game has a saddle point (a pure strategy solution). The value at that cell is the Value of the Game.

Mixed Strategy (No Saddle Point):

When $\maximin \neq \minimax$, no pure strategy exists. Players must randomize — choose strategies with certain probabilities.

For a 2×2 matrix with entries $[a, b; c, d]$:

- $p_1 = (d - c) / (a - b - c + d)$ — probability A plays row 1
- $q_1 = (d - b) / (a - b - c + d)$ — probability B plays column 1
- Value $V = (ad - bc) / (a - b - c + d)$

Dominance Method (for larger matrices):

- A strategy dominates another if it is at least as good in every outcome and strictly better in at least one.
- Eliminate dominated strategies row by row (for Player A, higher is better) and column by column (for B, lower is better).
- Repeat until the matrix reduces to a 2×2 or a saddle point emerges.

Sequencing Problem — Theory (n Jobs on m Machines)

Q50. [10 Marks] (Asked: Dec2020, Dec2023, Dec2024, May2022)

Solve the following sequencing problem of 4 Jobs on 6 Machines. (General Theory of Sequencing)

Answer:

A sequencing problem determines the optimal processing order of n jobs through m machines to minimize total elapsed time and idle time.

Johnson's Algorithm (n Jobs, 2 Machines):

- **Step 1:** List all jobs with their processing times on Machine 1 (M1) and Machine 2 (M2).
- **Step 2:** Find the job with the minimum processing time across all machines.
- **Step 3:** If the minimum is on M1, schedule that job first. If on M2, schedule it last.
- **Step 4:** Remove the selected job. Repeat Steps 2–3 with remaining jobs, filling from both ends.
- **Step 5:** Once the sequence is determined, compute the total elapsed time and idle time using a Gantt-style table.

Extension to n Jobs on m Machines:

For m machines ($m > 2$), Johnson's algorithm can be applied if either: (a) the minimum time on M1 $\geq$ maximum time on any middle machine, or (b) the minimum time on Mm $\geq$ maximum time on any middle machine. Combine M1 with middle machines as a fictitious machine G and Mm with middle machines as fictitious machine H, then apply Johnson's algorithm on G and H.

Key Definitions:

- **Total Elapsed Time:** Total time from start of first job on M1 to end of last job on Mm.
- **Idle Time on Machine:** Periods when a machine has no job to process, waiting for the previous machine to finish.
- **In-Process Waiting Time:** Time a job waits between finishing one machine and starting the next.

Monte Carlo Simulation — Detailed

Q51. [10 Marks] (Asked: Dec2020, Dec2021)

Explain the steps involved in Monte Carlo Simulation in detail.

Answer:

Monte Carlo Simulation is a powerful tool in OR for analyzing complex, stochastic (probabilistic) systems. It was developed during World War II and named after the Monte Carlo casino for its reliance on random chance.

Detailed Steps:

- **Step 1 – Problem Definition:** Clearly define the system and identify which variables are uncertain (random).
- **Step 2 – Data Collection:** Gather historical data or estimate probabilities for each uncertain variable. For example, daily demand, service time, etc.
- **Step 3 – Probability Distribution:** Construct a frequency/probability distribution table for each random variable.
- **Step 4 – Cumulative Distribution:** Compute cumulative probabilities (running total from 0 to 1).
- **Step 5 – Random Number Mapping:** Assign a range of two-digit random numbers (00–99) to each value in proportion to its probability. For example, if $P(\text{demand}=10) = 0.20$, assign random numbers 00–19.
- **Step 6 – Random Number Generation:** Use a random number table (or formula) to generate a sequence of random numbers.
- **Step 7 – Simulation Runs:** For each random number, read the corresponding variable value from the mapping table and record the system outcome (e.g., units sold, waiting time, profit).
- **Step 8 – Aggregate Results:** After sufficient runs (e.g., 50 or 100 trials), compute averages, totals, or statistics of interest.
- **Step 9 – Draw Conclusions:** Interpret results to make decisions: optimal inventory level, optimal number of servers, expected profit, etc.

Advantages:

- Applicable to problems too complex for exact mathematical analysis.
- Can model any probability distribution, not just standard ones.
- Requires no restrictive assumptions about the system.

Limitations:

- Results are approximate and depend on the number of simulation runs.
- Does not guarantee the optimal solution — only provides estimates.
- Requires reliable probability data; poor inputs give poor outputs (garbage-in, garbage-out).

Critical Path — Full Method

Q52. [10 Marks] (Asked: Dec2020, Dec2021, Dec2022, Dec2023, Dec2024)

Find the critical path for the following network. (General Theory)

Answer:

The Critical Path Method (CPM) is used to determine the minimum project duration and identify activities that must be closely monitored.

Procedure:

- **Step 1 – Network Construction:** Draw the project network: nodes represent events (milestones); arrows represent activities with their durations.
- **Step 2 – Forward Pass (Earliest Times):** Start from node 1 with $E(1) = 0$. For each node j : $E(j) = \max \text{ over all } i [E(i) + \text{duration}(i \rightarrow j)]$.
- **Step 3 – Backward Pass (Latest Times):** Start from the last node n with $L(n) = E(n)$. For each node i : $L(i) = \min \text{ over all } j [L(j) - \text{duration}(i \rightarrow j)]$.
- **Step 4 – Float Calculation:** For activity $i \rightarrow j$: Total Float = $L(j) - E(i) - \text{duration}$. Free Float = $E(j) - E(i) - \text{duration}$.

- **Step 5 – Identify Critical Path:** Mark all activities with Total Float = 0. Trace the path of critical activities from start to end — this is the Critical Path.
- **Step 6 – Project Duration:** The earliest completion time of the last event $E(n)$ = project duration.

Significance:

- Critical activities cannot be delayed without delaying the project.
- Non-critical activities have float — they can be delayed within limits without affecting project completion.
- CPM enables time-cost trade-off: crashing critical activities by spending more reduces project duration.

— End of UCC5C Theory Answer Bank —