

May 2023

17) Act

	Pay off in Rs.	
	Cold	Hot
Sell cold drinks	50	100
Sell hot drinks	120	40

The probability of hot weather is 0.8.
Prepare Opportunity loss table & select best act.

Sol

50	100
120	40

Step 1:- Identify Maximum payoffs.

hot drinks = Rs. 120

cold drinks = Rs. 100

Step 2:- Calculate Regret (maximum payoff - Actual payoff)

Act	Cold	Hot
Sell Cold	$120 - 50 = 70$	$100 - 100 = 0$
Sell hot	$120 - 120 = 0$	$100 - 40 = 60$

Step 3:- Calculate Expected Opportunity Loss table: [EOL]

$EOL = \sum (\text{Regret} \times \text{probability})$. here probability
hot = 0.8
cold = 0.2

$$EOL(\text{cold}) = 70(0.2) + 0(0.8) = 14$$

$$EOL(\text{hot}) = 0(0.2) + 60(0.8) = 48$$

The best act is 14. (minimum opportunity loss)

19) Given below is a pay off table from a reg.

EOL:-

states of nature

E_1

E_2

E_3

A_1

A_2

A_3

156

153

150

156

158

155

156

158

160

Step 1 :- Identify Maximum Payoff.

$$E_1 = 156$$

$$E_2 = 158$$

$$E_3 = 160$$

Step 2 :- Calculate

156-156	156-153	156-150
158-156	158-158	158-155
160-156	160-158	160-150

0	3	6
2	0	1
4	2	0

Dec - 2020

13) dual of primal lp problem

$$\text{Min } Z = 4x_1 + 5x_2 - 3x_3$$

Sub to,

$$\begin{aligned}
 x_1 + x_2 + x_3 &= 22 & \begin{cases} x_1 + x_2 + x_3 \geq 22 \\ x_1 + x_2 + x_3 \leq 22 \end{cases} \\
 3x_1 + 5x_2 - 2x_3 &\leq 65 \\
 x_1 + 7x_2 + 4x_3 &\leq 120 \\
 x_1 \geq 0, x_2 > 0 \text{ \& } x_3
 \end{aligned}$$

Sol

Primal:-

$$\text{Min } Z = 4x_1 + 5x_2 - 3x_3$$

Sub to,

$$\begin{aligned}
 x_1 + x_2 + x_3 &\geq 22 \\
 -x_1 - x_2 - x_3 &\geq -22 \\
 -3x_1 - 5x_2 + 2x_3 &\geq -65 \\
 -x_1 - 7x_2 - 4x_3 &\geq -120
 \end{aligned}$$

$$x_1, x_2 \geq 0, x_3 \text{ is unrestricted.}$$

$$\text{Let } x_3 = x_3' - x_3'' \text{ (Converting } x_3)$$

$$\text{Min } Z = 4x_1 + 5x_2 - 3x_3' + 3x_3''$$

Sub to

$$\begin{aligned}
 &x_1 + x_2 + x_3' - x_3'' \geq 22 \\
 &-x_1 - x_2 - x_3' + x_3'' \geq -22 \\
 &-3x_1 - 5x_2 + 2x_3' - 2x_3'' \geq -65 \\
 &-x_1 - 7x_2 - 4x_3' + 4x_3'' \geq -120
 \end{aligned}$$

$$x_1, x_2, x_3, x_4 \geq 0$$

4 Constraints = 4 Variables introduced $\{y_1, y_2, y_3 \text{ \& } y_4\}$

$$\text{Dual Max } Z = 22y_1 - 22y_2 - 65y_3 - 120y_4$$

Sub to

$$y_1 - y_2 - 3y_3 - y_4 \leq 4$$

$$y_1 - y_2 - 5y_3 - 7y_4 \leq 5$$

$$y_1 - y_2 + 2y_3 - 4y_4 \leq -3$$

$$-y_1 + y_2 - 2y_3 + 4y_4 \leq 3$$

$$22(y_1 - y_2) = y_1'$$

$$\text{Max } Z = 22y_1' - 65y_3 - 120y_4$$

Sub to,

$$y_1' - 3y_3 - y_4 \leq 4$$

$$y_1' - 5y_3 - 7y_4 \leq 5$$

$$y_1' + 2y_3 - 4y_4 \leq -3$$

$$-y_1' + 2y_3 + 4y_4 \leq 3$$

$$y_3, y_4 \geq 0, y_1' \text{ is unrestricted}$$

x

Dec 2022

16) Write down the dual of

$$\text{Min } Z = 7x_1 + 10x_2$$

Sub to,

$$10x_1 + 13x_2 \geq 50$$

$$7x_1 + 9x_2 \leq 40$$

$$3x_1 + 5x_2 = 20$$

$$x_1 + x_2 \geq 0$$

$$3x_1 + 5x_2 \leq 20$$

$$3x_1 + 5x_2 \geq 20$$

$$\text{Min } Z = 7x_1 + 10x_2$$

sub to,

$$10x_1 + 13x_2 \geq 50$$

$$7x_1 + 9x_2 \leq 40$$

$$3x_1 + 5x_2 \geq 20$$

$$3x_1 + 5x_2 \leq 20$$

$$x_1, x_2 \geq 0$$

Page No. _____
Date _____

Primal Problem:

$$\text{Min } Z = 7x_1 + 10x_2$$

Sub to

$$10x_1 + 13x_2 \geq 50$$

$$-7x_1 - 9x_2 \geq -40$$

$$3x_1 + 5x_2 \geq 20$$

$$-3x_1 - 5x_2 \geq -30$$

10	13
-7	-9
3	5
-3	-5

$$x_1, x_2 \geq 0.$$

Dual Problem:

4 Constraints So, Introduce 4 Variables
Dual Variable $\leftarrow (y_1, y_2, y_3, y_4)$

$$\text{Max } Z = 50y_1 - 40y_2 + 20y_3 - 20y_4$$

Sub to the Constraints,

$$10y_1 - 7y_2 + 3y_3 - 3y_4 \leq 7$$

$$13y_1 - 9y_2 + 5y_3 - 5y_4 \leq 10$$

$$y_1, y_2, y_3, y_4 \geq 0.$$

Substitute

Let's change $[y_3 - y_4 = y_3']$.

$$\text{Max } Z = 50y_1 - 40y_2 + 20y_3'$$

Sub to

$$10y_1 - 7y_2 + 3y_3' \leq 7$$

$$13y_1 - 9y_2 + 5y_3' \leq 10$$

$y_1, y_2 \geq 0$, y_3' is unrestricted Variables.

May 2023

15) Dual primal:

$$\text{Min } Z = 3x_1 - 2x_2 + x_3$$

Sub to,

$$2x_1 - 3x_2 + x_3 \leq 5$$

$$4x_1 - 2x_2 \geq 9$$

$$-8x_1 + 4x_2 + 3x_3 = 8$$

$x_1, x_2 \geq 0$, x_3 is unrestricted.

Sol Primal:

$$\text{Min } Z = 3x_1 - 2x_2 + x_3$$

Sub to

$$2x_1 - 3x_2 + x_3 \leq 5$$

$$4x_1 - 2x_2 \geq 9$$

$$-8x_1 + 4x_2 + 3x_3 \leq 8$$

$$-8x_1 + 4x_2 + 3x_3 \geq 8$$

$x_1, x_2 \geq 0$, x_3 is unrestricted.

$$\text{let } x_3 = x_3' - x_3''$$

Dual

min

$$\text{Max } Z = 3x_1 - 2x_2 + x_3' - x_3''$$

Sub to

$$-2x_1 + 3x_2 - x_3' + x_3'' \geq -5$$

$$\begin{bmatrix} -2 & 3 & -1 & 1 \\ 4 & -2 & 0 & 0 \\ 8 & -4 & -3 & 3 \\ -8 & 4 & 3 & -3 \end{bmatrix}$$

$$4x_1 - 2x_2 + 0x_3' - 0x_3'' \geq 9$$

$$8x_1 - 4x_2 - 3x_3' + 3x_3'' \geq -8$$

$$-8x_1 + 4x_2 + 3x_3' - 3x_3'' \geq 8$$

$$x_1, x_2, x_3', x_3'' \geq 0$$

Dual :- 4 variables = 4 constraints (variables y_1, y_2, y_3, y_4)

$$\text{Max } Z = -5y_1 + 9y_2 - 8y_3 + 8y_4$$

Sub to the constraints,

$$-2y_1 + 4y_2 + 8y_3 - 8y_4 \leq 3$$

$$3y_1 - 2y_2 - 4y_3 + 4y_4 \leq -2$$

$$-y_1 + 3y_3 + 3y_4 \leq 1$$

$$y_1 + 3y_3 - 3y_4 \leq -1$$

$$y_1, y_2, y_3, y_4 \geq 0$$

Substitute $y_3' = y_3 - y_4$

$$\text{Max } Z = -5y_1 + 9y_2 - 8y_3'$$

Sub to the constraints,

$$-2y_1 + 4y_2 + 8y_3 \leq 3$$

$$3y_1 - 2y_2 - 4y_3 \leq -2$$

$$-y_1 - 3y_3 \leq 1$$

$$y_1 + 3y_3 \leq -1$$

① Use duality & solve.

$$\text{Min } Z = 3x_1 + 5x_2$$

Sub to

$$x_1 + x_2 \geq 200$$

$$x_1 \leq 80$$

$$x_2 \geq 60$$

$$x_1, x_2 \geq 0$$

Sol

Primal

$$\text{Min } Z = 3x_1 + 5x_2$$

Sub to

$$x_1 + x_2 \geq 200$$

$$-x_1 + 0x_2 \geq -80$$

$$0x_1 + x_2 \geq 60$$

$$x_1, x_2 \geq 0$$

$$\begin{bmatrix} 1 & 1 \\ -1 & 0 \\ 0 & 1 \end{bmatrix}$$

Dual Let y_1, y_2, y_3 be the dual variable

$$\text{Max } Z = 200y_1 - 80y_2 + 60y_3$$

Sub to

$$1y_1 - 1y_2 + 0y_3 \leq 3$$

$$1y_1 + 0y_2 + 1y_3 \leq 5$$

$$y_1, y_2, y_3 \geq 0$$

Solve using simplex.

slack
 $(\leq + S_1)$
 $(\geq - S_1)$
 surplus

$$\text{Max } Z = 200y_1 - 80y_2 + 60y_3 + 0S_1 + 0S_2$$

Sub to

$$y_1 - y_2 + 0y_3 + S_1 = 3$$

$$y_1 + 0y_2 + y_3 + S_2 = 5$$

put $y_1 = y_2 = y_3 = 0$

Table no 1: $S_1 = 3$ } Basic Variable.
 $S_2 = 5$

Table 1

			C _j					Min Ratio
			200	-80	60	0	0	
			y ₁	y ₂	y ₃	S ₁	S ₂	
C _B	B	x _B	Key Column					
			y ₁	y ₂	y ₃	S ₁	S ₂	
0	S ₁	3	1	-1	0	1	0	$\frac{3}{1} = 3$
0	S ₂	5	1	0	1	0	1	$\frac{5}{1} = 5$
Z _j			0	0	0	0	0	
(C _B - Y _j)								
Z _j - C _j			-200	80	-6	0	0	

y₁ → Entering Variable.

S₁ → leaving Variable.

→ Highest Negative number

$$\text{New } y_1 = \left(\frac{S_1}{\text{key element}} \right) = 1.$$

$$\text{New } y_1 = \frac{3}{1}, \frac{1}{1}, \frac{-1}{1}, \frac{0}{1}, \frac{1}{1}, \frac{0}{1} = 3, 1, -1, 0, 1, 0$$

$$\text{New } S_2 = \text{Old } S_2 - \text{Key no} (\text{New } y_1).$$

$$\text{old } S_2 = 5 \quad 1 \quad 0 \quad 1 \quad 0 \quad 1$$

$$+ \text{New } y_1 \quad -3 \quad -1 \quad 1 \quad 0 \quad -1 \quad 0$$

$$2 \quad 0 \quad -1 \quad 1 \quad -1 \quad 1$$

Table: 2

C_B	C_j	x_B	200	80	60	0	0	min ratio (÷ve).
	B		y_1	y_2	y_3	S_1	S_2	
200	y_1	3	1	-1	0	1	0	$\frac{3}{-1} = -3$
0	S_2	2	0	1	1	-1	1	$\frac{2}{1} = 2$

Key row Z_j 200 -200 0 200 0

$Z_j - Z_j$ 0 -120 -60 200 0

key column

$y_2 \rightarrow$ Entering Variable.

$S_2 \rightarrow$ leaving Variable.

$$\text{New } y_1 = \text{old } y_2 - \text{key no} (\text{new } y_2)$$

$$= 3 \quad 1 \quad -1 \quad 0 \quad 1 \quad 0$$

$$+ 200 \quad 1 \quad 1 \quad -1 \quad 1$$

$$5 \quad 0 \quad 0 \quad 1 \quad 0 \quad 1$$

Table 3

C_j			200	-80	60	0	0
C_B	B	x_B	y_1	y_2	y_3	s_1	s_2
200	y_1	5	1	0	1	0	1
-80	y_2	2	0	1	1	-1	1
Z_j			200	-80	120	80	120
$Z_j - C_j$			0	0	60	80	120

Since all $Z_j - C_j \geq 0$,

$$y_1 = 5, y_2 = 2, y_3 = 0$$

$$\text{Max } Z = 200y_1 - 80y_2 + 60y_3$$

$$= 200(5) - 80(2)$$

$$= 1000 - 160$$

$$\text{Max } Z = 840$$

$$x_1 + x_2 = 200, \quad \boxed{x_1 = 80} \quad \text{Min } Z = 3x_1 + 5x_2$$

$$80 + x_2 = 200$$

$$x_2 = 200 - 80$$

$$\boxed{x_2 = 120}$$

$$= 3(80) + 5(120)$$

$$= 240 + 600$$

$$\boxed{\text{Min } Z = 840}$$

Simplex Method.

Solve by L.P.P. Max $Z = 3x + 2y$.

Subject to,

$$2x + y \leq 40,$$

$$x + y \leq 24$$

$$2x + 3y \leq 60; x, y \geq 0.$$

Sol

Max $Z = 3x + 2y + 0s_1 + 0s_2 + 0s_3$
Sub to,

$$2x + y + s_1 = 40$$

$$x + y + s_2 = 24$$

$$2x + 3y + s_3 = 60$$

put $x=y=0$, $s_1=40$, $s_2=24$, $s_3=60$. is initial basic feasible solution.

Table 1

C_j			3	2	0	0	0	$x_B \div \text{Key column}$
C_B	B	x_B	x	y	s_1	s_2	s_3	Min (+ve) Ratio.
0	s_1	40	2	1	1	0	0	$40/2 = 20$
0	s_2	24	1	1	0	1	0	$24/1 = 24$
0	s_3	60	2	3	0	0	1	$60/2 = 30$

$$Z_j = C_B x_i \quad 0 \quad 0 \quad 0 \quad 0 \quad 0$$

$$Z_j - C_j \quad -3 \quad -2 \quad 0 \quad 0 \quad 0$$

Entering Variable = x ; Leaving Variable = s_1 ;
Pivot Element = 2.

$$\frac{40}{2}, \frac{2}{2}, \frac{1}{2}, \frac{1}{2}, \frac{0}{2}, \frac{0}{2}$$

Entering Variable = y
 Leaving Variable = S₂
 Pivot element = $\frac{1}{2}$

Table: 2

C _B	C _j	X _B	x	y	S ₁	S ₂	S ₃	X _B Key no
3	x	20	1	$\frac{1}{2}$	$\frac{1}{2}$	0	0	Min (fre) Ratio $\frac{20}{1/2} = 20 \times 2 = 40$
0	S ₂	4	0	$\frac{1}{2}$	$-\frac{1}{2}$	1	0	$\frac{4}{1/2} = 4 \times 2 = 8$
0	S ₃	~~20~~ 20	0	$\frac{1}{2}$	$-\frac{1}{2}$	0	1	$\frac{20}{1/2} = 10 \times 2 = 20$
Z _j		60	3	$\frac{3}{2}$	$\frac{3}{2}$	0	0	
Z _j - C _j			0	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	

New S₂ = Old S₂ - Key no (new x)

$$\begin{aligned}
 &24 \quad 1 \quad 1 \quad 0 \quad 1 \quad 0 \quad 24 \\
 \Rightarrow &20 \quad 1 \quad \frac{1}{2} \quad \frac{1}{2} \quad 0 \quad 0 \\
 &4 \quad 0 \quad \frac{1}{2} \quad -\frac{1}{2} \quad 1 \quad 0
 \end{aligned}$$

$$\text{New } S_2 = \text{Old } S_3 - \text{Key no (new x)}$$

$$60 \quad 2 \quad 3 \quad 0 \quad 0 \quad 1$$

$$20 \quad 1 \quad \frac{1}{2} \quad \frac{1}{2} \quad 0 \quad 0 \quad \times (-2)$$

New S_3

$$\begin{array}{r}
 60 \\
 40 \\
 \hline
 (-) \quad 20 \quad 0 \quad 2 \quad -1 \quad 0 \quad 1
 \end{array}$$

Table 3

C_B	C_j	x_B	x	y	S_1	S_2	S_3	Min (Max) Ratio
3	x	16	1	0	1	-1	0	
2	y	8	0	1	-1	2	0	
0	S_3	4	0	0	1	-4	1	

$$\begin{array}{l}
 Z_j: (16 \times 3) + (8 \times 2) = 64 \\
 Z_j - C_j: 0 \quad 0 \quad 1 \quad -1 \quad 0 \\
 \text{New } y = \text{Old } y - \text{Key} \times \text{new } y
 \end{array}$$

$$\begin{array}{r}
 4 \quad 0 \quad \frac{1}{2} \quad -\frac{1}{2} \quad 1 \quad 0 \\
 \times 2 \\
 8 \quad 0 \quad 1 \quad -1 \quad 2 \quad 0
 \end{array}$$

$$\text{New } x = \text{Old } x - \text{Key} \times (\text{new } y)$$

$$20 \quad 1 \quad \frac{1}{2} \quad \frac{1}{2} \quad 0 \quad 0$$

$$4 \quad 0 \quad \frac{1}{2} \quad -\frac{1}{2} \quad 1 \quad 0$$

$$(-) \quad 16 \quad 1 \quad 0 \quad 1 \quad -1 \quad 0$$

$$Z = 64 ; x = 16 ; y = 8 ;$$

8 0 1 -1 2 0 (x 2)

Page No.	
Date	

$$\text{New } S_3 = \text{Old } S_3 - \text{Keynd}(\text{new } y).$$

0 0 20 0 2 4 1 0 1

1 0 16 0 2 ~~2~~ 4 0

(-)

4 0 0 1 -4 1

Final Answer

$$Z = 64; x = 16; y = 8.$$